ISSN 2229-5518

Chemical Reaction Effects on Radiative MHD Oscillatory Flow in a Porous Channel with Mass Transfer and Heat absorption

R. Vijayalakshmi, A. Govindarajan, J.Sasi kumar, E.P.Siva

Abstract— An attempt has been made to study the chemical reaction effects on radiative MHD oscillatory flow in a porous channel with mass transfer and heat absorption in a symmetric channel. On the basis of certain assumptions, the momentum, energy, concentration equations are obtained. The governing equations of MHD oscillatory flow were non-dimensionalised, simplified and solved. The closed analytical form solutions for velocity, temperature and concentration were obtained. The numerical computations were presented graphically to show the salient features of the physical parameters Peclet number, Grashof number, Reynolds number, Hartmann number, porous medium shape factor. The skin friction, Nusselt number and Sherwood number were also presented graphically and analyzed qualitatively.

Index Terms— chemical reaction, heat absorption, mass transfer, MHD oscillatory flow, Nusselt number, porous channel, skin friction, Sherwood number, thermal radiation,

1 Introduction

MHD oscillatory flow plays an important role in petroleum industries, exploration and thermal recovery of oil, underground nuclear waste storage sites, geophysics, astrophysics, aeronautics, electronics, chemical engineering etc. In the field of power generation, MHD flow receives considerable attention because it offers higher thermal efficiencies in power plants. The study of MHD oscillatory flow in a porous channel has been receiving considerable attention due to its applications in soil mechanics, ground water hydrology, irrigation, drainage, water purification processes, absorption and filtration processes in chemical engineering. The effects of combined heat and mass transfer with chemical reaction of MHD oscillatory flow have many applications in design of chemical processing equipments, formation and dispersion of FLOG, food processing and cooling of tower.

Soundalgekar [1] studied the MHD oscillatory flow past a semi-infinite plate. Singh et. al. [2] discussed the MHD oscillatory flow with heat transfer in a flat plate. In 1985, Ramachandra Rao et. al. [3] made a detailed study of the MHD oscillatory flow of blood through channels of variable cross section. Gholizadeh [4] investigated the MHD oscillatory flow through porous medium. Singh [5] numerically found an exact solution of an oscillatory MHD flow in a channel filled with porous medium. Later, Makinde [6] considered the MHD oscillatory flow of a porous medium channel.

- R. Vijayalakshmi, SRM University, Ramapuram, Chennai, India., E-mail: vijayalakshmi.r@rmp.srmuniv.ac.in
- A. Govindarajan, SRM University, Kattankulathur, Chennai, India. E-mail: govindarajan.a@ktr.srmuniv.ac.in

The main aim of this paper is to investigate the MHD oscillatory flow of an electrically conducting, chemically reacting optically thin fluid through a symmetric channel with non-uniform wall temperatures. The governing equations of fluid flow are solved subject to relevant boundary conditions. The influence of several pertinent parameters on velocity, temperature, concentration, Nusselt number, skin friction, Sherwood number has been studied and numerical results obtained are presented graphically. The problem is formulated in Section 2. Section 3 comprises the solutions for flow, heat and mass transfer analysis. The graphical results are presented and discussed in Section 4. Section 5 contains the final remarks.

2 Mathematical Analysis

Consider the flow of an electrically conducting, heat generating, optically thin and chemically reacting oscillatory fluid in a symmetric channel filled with saturated porous medium under the influence of an externally applied homogeneous magnetic field and radiative heat transfer as shown in Fig. 1.

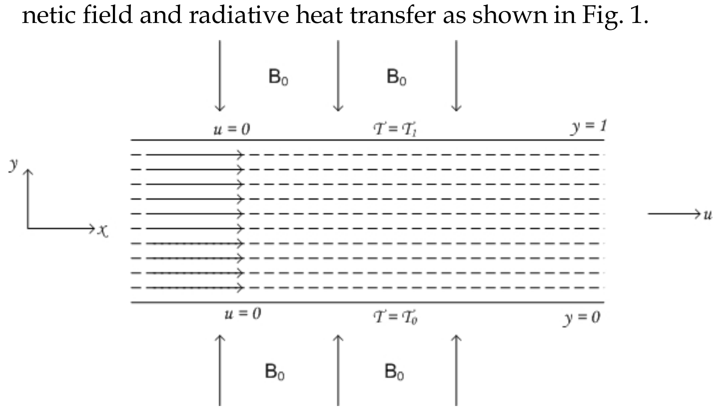

Fig.1 Schematic diagram of the physical model

ISSN 2229-5518

Take a Cartesian coordinate system $(\bar{x}, \bar{y})$ where $\bar{x}$ lies along the centre of the channel, $\bar{y}$ is the distance measured in the normal section. The walls of the channel are maintained at temperatures T_0 and T_1 respectively, with heat source/sink parameter α. C_1 and C_2 are the species concentrations at the walls with mass diffusion coefficient D_m. The fluid is assumed to absorb its own emitted radiation. It is also assumed that the transversely applied magnetic field and magnetic Reynolds number are very small and hence the induced magnetic field is negligible. Viscous and Darcy's resistance terms are taken into account with constant permeability of the porous medium. Under these assumptions, the governing equations for a MHD oscillatory flow in a symmetric channel with Boussinesq approximation may be written as:

Momentum equation

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} - \frac{\nu}{k}u - \frac{\sigma B_0^{\,2} u}{\rho} + g\beta\left(T - T_0\right) + g\beta^*\left(C - C_0\right) \quad (1)$$

Energy equation

$$\frac{\partial T}{\partial t} = \frac{K}{\rho C_P}\frac{\partial^2 T}{\partial y^2} + \frac{1}{\rho C_P}\frac{\partial q}{\partial y} + \frac{Q}{\rho C_P} \quad (2)$$

Concentration equation

$$\frac{\partial C}{\partial t} = D_m\frac{\partial^2 C}{\partial y^2} - K_c{}'\left(C - C_0\right) \quad (3)$$

together with the boundary conditions

$$u = 0;\ T = T_1;\ C = C_1 \ \text{ on } \ y = 1 \quad (4)$$

$$u = 0;\ T = T_0;\ C = C_0 \ \text{ on } \ y = 0 \quad (5)$$

Since the fluid is optically thin with relatively low density, then according to Ogulu and Bestman [7] the radiative heat flux is given by

$$\frac{\partial q}{\partial y} = 4\alpha^2\left(T - T_0\right) \quad (6)$$

To non-dimensionalize the governing equations, the following dimensionless variables were introduced.

$$\bar{x} = \frac{x}{a};\ \bar{y} = \frac{y}{a};\ \bar{u} = \frac{u}{U};\ \mathrm{Re} = \frac{U a}{\nu};\ \theta = \frac{T - T_0}{T_1 - T_0};$$

$$\phi = \frac{C - C_0}{C_1 - C_0};\ M^2 = \frac{\sigma B_0^{\,2} a^2}{\rho\nu};\ \bar{t} = \frac{tU}{a};\ \bar{P} = \frac{P\,a}{\rho\nu U};$$

$$Da = \frac{k}{a^2};\ s^2 = \frac{1}{D_a};\ Sc = \frac{U a}{D_m};\ Gr = \frac{g\beta\left(T_1 - T_0\right)a^2}{\nu U};$$

$$Gc = \frac{g\beta^*\left(C_1 - C_0\right)a^2}{\nu U};\ Pe = \frac{U a\rho\, C_P}{K};\ N^2 = \frac{4\alpha^2 a^2}{K};$$

$$K_c = \frac{K_c{}'\,a}{U};\ \alpha = \frac{Q\,a^2}{K\left(T_1 - T_0\right)} \quad (7)$$

3 Solution of the Problem

The dimensionless governing equations together with the appropriate boundary conditions (after removing bars) can be written as

$$\mathrm{Re}\frac{\partial u}{\partial t} = -\frac{\partial P}{\partial x} + \frac{\partial^2 u}{\partial y^2} - \left(s^2 + M^2\right)u + Gr\,\theta + Gc\,\phi \quad (8)$$

$$Pe\frac{\partial\theta}{\partial t} = \frac{\partial^2\theta}{\partial y^2} + N^2\,\theta + \alpha \quad (9)$$

$$\frac{\partial\phi}{\partial t} = \frac{1}{Sc}\frac{\partial^2\phi}{\partial y^2} - K_C\,\phi \quad (10)$$

with $u = 0;\ \theta = 1;\ \phi = 1$ on $y = 1$ (11)

and $u = 0;\ \theta = 0;\ \phi = 0$ on $y = 0$ (12)

For a purely oscillatory flow, we take the pressure gradient of the form $-\frac{\partial P}{\partial x} = \lambda e^{i\omega t}$

where λ is constant and ω is the frequency of oscillations. Due to the selected form of pressure gradient, we assume that the solutions for $u(y,t)$, $\theta(y,t)$ and $\phi(y,t)$ be in the form

$$u(y,t) = u_0(y)\,e^{i\omega t};\ \theta(y,t) = \theta_0(y)\,e^{i\omega t} \text{ and } \phi(y,t) = \phi_0(y)\,e^{i\omega t} \quad (12)$$

Substituting Eq. (12) in Eqs. (8), (9), (10), we obtain

$$\frac{d^2\theta_0}{\partial y^2} + m^2\,\theta_0 = -\alpha\,e^{-i\omega t} \quad (13)$$

$$\frac{d^2\phi_0}{\partial y^2} - Sc\,Y^2\,\phi_0 = 0 \quad (14)$$

$$\frac{d^2 u_0}{\partial y^2} - n^2\,u_0 = -\lambda - Gr\,\theta_0 - Gc\,\phi_0 \quad (15)$$

together with the boundary conditions

$u = 0;\ \theta = 1;\ \phi = 1$ on $y = 1$ (16)

and $u = 0;\ \theta = 0;\ \phi = 0$ on $y = 0$ (17)

where $m = \sqrt{N^2 - i\,\omega\,Pe}$, $Y = \sqrt{K_c + i\,\omega}$ and $n = \sqrt{s^2 + M^2 + i\,\omega\,\mathrm{Re}}$.

Equations (13), (14), (15) are solved using Eqs. (16) and (17). We obtain

$$\theta(y,t) = \left(X\cos(m\,y) + \frac{1 + X\left(1 - \cos m\right)}{\sin m}\sin(m\,y) - X\right)e^{i\omega t} \quad (18)$$

$$\phi(y,t) = \left(\frac{\sinh\left(\sqrt{Sc}\ \,Y\right)\,y}{\sinh\left(\sqrt{Sc}\ \,Y\right)}\right)e^{i\omega t} \quad (19)$$

$$u(y,t)=\left(\begin{array}{l}\left(\dfrac{\left(\dfrac{\lambda}{n^2}-\dfrac{Gr\,X}{n^2}\right)\left(e^{-n}-1\right)+\dfrac{Gr}{m^2+n^2}\left[X\,e^{-n}-2X-1+X\cos m\right]+\dfrac{Gc}{ScY^2-n^2}}{2\sinh n}\right)e^{ny}\\ +\left(\dfrac{\left(\dfrac{\lambda}{n^2}-\dfrac{Gr\,X}{n^2}\right)\left(1-e^{n}\right)-\dfrac{Gr}{m^2+n^2}\left[X\,e^{n}-2X-1+X\cos m\right]-\dfrac{Gc}{ScY^2-n^2}}{2\sinh n}\right)e^{-ny}\\ +\left(\dfrac{\lambda}{n^2}-\dfrac{Gr\,X}{n^2}\right)+\dfrac{Gr}{m^2+n^2}\left[X+\dfrac{\sin(m\,y)}{\sin m}\left(1+X-X\cos m\right)\right]\\ -\dfrac{Gc}{ScY^2-n^2}\cdot\dfrac{\sinh\left(\sqrt{Sc}\,Y\right)y}{\sinh\left(\sqrt{Sc}\,Y\right)}\end{array}\right)e^{i\omega t} \quad (20)$$

where $X=\dfrac{\alpha}{m^2}e^{-i\omega t}$.

Skin friction
The skin friction (shear stress) at the wall is given by

$$\tau=\left[\mu\frac{\partial u}{\partial y}\right] at\ y=0, y=1\,. \quad (21)$$

On simplification, we get

$$\tau=\mu\left(\begin{array}{l}n\left(\dfrac{\left(\dfrac{\lambda}{n^2}-\dfrac{Gr\,X}{n^2}\right)\left(e^{-n}-1\right)+\dfrac{Gr}{m^2+n^2}\left[X\,e^{-n}-2X-1+X\cos m\right]+\dfrac{Gc}{ScY^2-n^2}}{2\sinh n}\right)e^{ny}\\ +(-n)\left(\dfrac{\left(\dfrac{\lambda}{n^2}-\dfrac{Gr\,X}{n^2}\right)\left(1-e^{n}\right)-\dfrac{Gr}{m^2+n^2}\left[X\,e^{n}-2X-1+X\cos m\right]-\dfrac{Gc}{ScY^2-n^2}}{2\sinh n}\right)e^{-ny}\\ +\dfrac{Gr}{m^2+n^2}\left[\dfrac{m\cos(m\,y)}{\sin m}\left(1+X-X\cos m\right)\right]\\ -\dfrac{Gc}{ScY^2-n^2}\cdot\dfrac{\left(\sqrt{Sc}\,Y\right)\cosh\left(\sqrt{Sc}\,Y\right)y}{\sinh\left(\sqrt{Sc}\,Y\right)}\end{array}\right)e^{i\omega t}\ at\ \ y=0, y=1 \quad (22)$$

Nusselt Number
The rate of heat transfer across the channel is given by

$$Nu=-\left[\frac{\partial\theta}{\partial y}\right] at\ y=0,\ y=1 \quad (23)$$

$$Nu=-\left(\begin{array}{l}(-m)X\sin(m\,y)\\ +m\left(\dfrac{1+X\left(1-\cos m\right)}{\sin m}\right)\cos(m\,y)\end{array}\right)e^{\,i\omega t}\quad at\ \ y=0,\ \ y=1 \quad (24)$$

Sherwood Number
The rate of mass transfer across the channel is given by

$$Sh=\left[\frac{\partial C}{\partial y}\right] at\ y=0, y=1 \quad (25)$$

$$Sh=\left(\frac{(\sqrt{Sc}\ \ Y)\ \cosh\ (\sqrt{Sc}\ \ Y)\ y}{\sinh\ (\sqrt{Sc}\ \ Y)}\right)e^{\,i\omega t}\quad at\ y=0, y=1 \quad (26)$$

4 Graphical Results and Discussions

To study the effects of chemical reaction, heat and mass transfer on the radiative MHD oscillatory flow in a symmetric channel, the velocity u , temperature θ and the species concentration C profiles are depicted graphically against y for different physical parameters: Grashof number for heat transfer, Grashof number for mass transfer, radiation parameter N, Hartmann number M, Reynolds number Re, Schmidt number Sc and chemical reaction parameter K_c. The graphs are plotted using MATLAB 6.5.

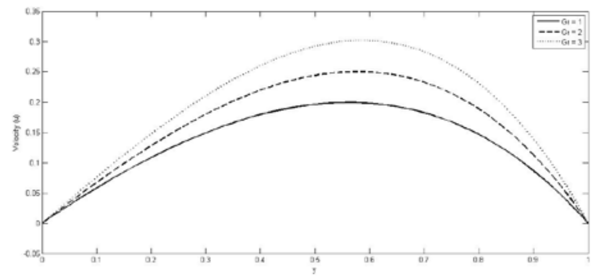

Fig. 2 Effect of Gr on Velocity

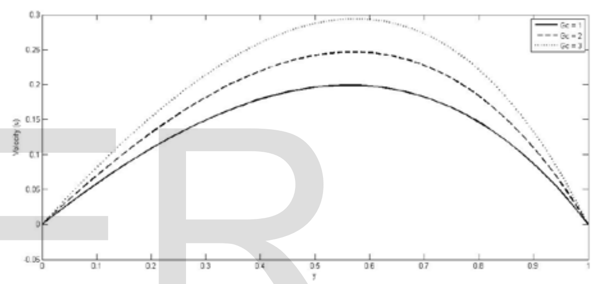

Fig. 3 Effect of Gc on Velocity

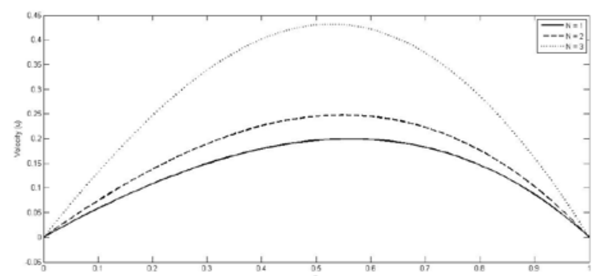

Fig. 4 Effect of radiation parameter N on Velocity

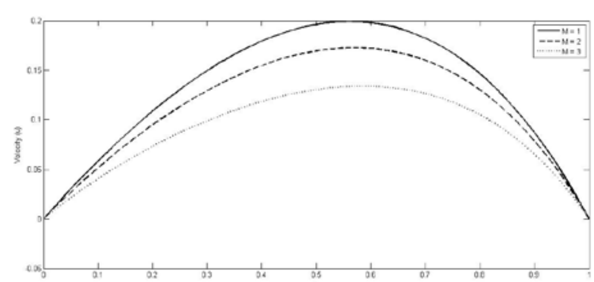

Fig. 5 Effect of Hartmann number M on Velocity

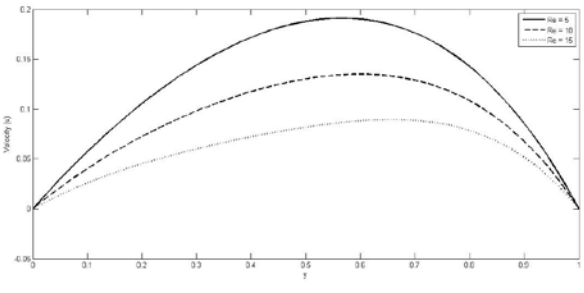

Fig. 6 Effect of Reynolds number Re on Velocity

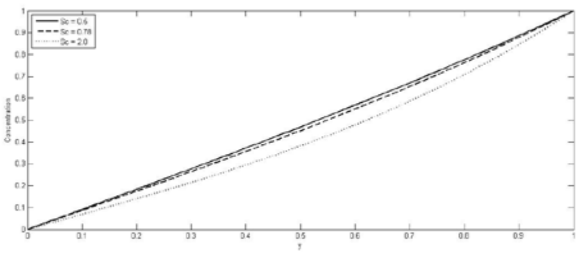

Fig. 7 Effect of Schmidt number Sc on Concentration

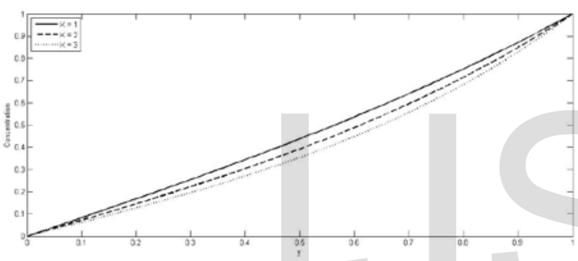

Fig. 8 Effect of Chemical reaction parameter Kc on Concentration

Fig. 2 and Fig. 3 show the effect of Grashof number Gr and Gc on velocity u. It is noted that as Grashof number increase, the velocity profiles increase. All the profiles start increasing steadily near the lower end of the asymmetric channel upto the midpoint of the channel, thereafter they start decreasing steadily at the upper end of the channel.

Fig. 4 shows the effect of radiation parameter N on the velocity profiles u. It is found that the velocity profiles increase steadily as the radiation parameter increases. All the profiles increase steadily from the lower plate and reach the maximum value at a point little away from the upper plate and thereafter they decrease steadily and reach the value zero at the upper plate.

Fig. 5 depicts the effect of Hartmann number M on the velocity profiles u. It is observed that the velocity decelerates when there is an increase in Hartmann number. The profiles retard continuously. This is due to Lorentz force.

Fig. 6 illustrates the effect of Reynolds number on the velocity profile u. It is seen that the velocity profiles decrease as Reynolds number Re increases.

The effect of Schmidt number on concentration profiles are shown in Fig. 7. The values of Sc are chosen as 0.5, 0.78 and 2.0, which correspond to Hydrogen gas, Ammonia and ethyl benzene in air respectively. It is clear that the concentration profiles decrease uniformly whenever there is an increase in Schmidt number Sc.

The effect of chemical reaction parameter K_c on the concentration profiles is plotted in Fig. 8. It is seen that the profiles decrease steadily whenever there is an increase in chemical reaction parameter K_c ($K_c > 0$). For $K_c < 0$, the profiles show a reverse trend. Due to the sake of brevity, it is not plotted.

5 Final Remarks

In this section, we studied the effect of heat and mass transfer with chemical reaction on the radiative MHD oscillatory flow in an asymmetric channel. The governing equations of momentum, energy and species concentration have been written in dimensionless form using dimensionless parameters. A closed form of analytical solution using Perturbation method has been employed to evaluate and to solve the dimensionless velocity u, the dimensionless temperature θ, the dimensionless species concentration C, skin friction τ, Nusselt number Nu and Sherwood number Sh. The main findings are summarized below:

- Decrease in Grashof number for heat transfer Gr and Grashof number for mass transfer Gc have accelerating effects on velocity of the flow field.
- Increase in Hartmann number M decreases the velocity of the flow field at all points, due to the magnetic pull of the Lorentz force acting on the flow field.
- Increase in the radiation parameter N and Reynolds number Re decrease the velocity of the flow field.
- Increase in Schmidt number Sc and chemical parameter K_c decrease the species concentration. Hence the consumption of chemical species causes a fall in the concentration field, which in turn diminishes the buoyancy effects due to concentration gradients.

When there is no chemical reaction and radiation effects, the results found in this section are in good agreement with the results obtained by Makinde and Mhone [6].

6 Nomenclature

$B_0 = \mu_e H_0$	electromagnetic induction
H_0	intensity of the magnetic field
C_P	specific heat at constant pressure
D_a	Darcy number
D_m	mass diffusion coefficient
g	gravitational force
Gr	Grashof number for heat transfer
Gc	Grashof number for mass transfer
M	Hartmann number
K	thermal conductivity
k	porous medium permeability coefficient
N	radiation parameter
P	pressure
q	radiative heat flux
s	porous medium shape factor
t	time
T	fluid temperature
u	axial velocity

ISSN 2229-5518

U	flow mean velocity
K_c	chemical reaction parameter
Q	heat absorption/generation at the channel
Pe	Peclet number
Re	Reynolds number
Sc	Schmidt number
Nu	Nusselt number
Sh	Sherwood number

Greek Symbols

θ	dimensionless fluid temperature function
β	coefficient of thermal expansion due to temperature
β^*	coefficient of thermal expansion due to concentration
μ_e	magnetic permeability
σ	conductivity of the fluid
ρ	fluid density
ν	kinematic viscosity coefficient
λ	wavelength
ω	frequency of the oscillation
α	heat source/sink parameter
τ	skin friction

REFERENCES

[1] V.M. Soundalgekar, H.S. Takhar, "MHD Oscillatory Flow Past a Semi-infinite Plate", AIAA Journal, Vol. 15, 1977, pp. 457-458

[2] M. Singh, V.M. Soundalgekar and T.V. Ramana Murty, "MHD Oscillatory Flow with Heat Transfer in a Flat Plate", Int. J. of Energy Research, Vol. 6, 1982, pp. 29-36.

[3] A. Ramachandra Rao, K.S. Deshikachar, "MHD Oscillatory Flow of Blood through Channels of Variable Cross Section", Int. J. Eng. Sci., Vol. 10, 1985, 1615-1628.

[4] A. Gholizadeh, "MHD Oscillatory Flow through Porous Medium", Astro-physics and Space Science, Vol. 180, 1991, pp. 287-292.

[5] K.D. Singh, "Exact Solution of an Oscillatory MHD Flow in a channel filled with porous medium", Int. J. of Applied Mechanics and Engineering, Vol. 16, 2011, pp. 277-283.

[6] O.D. Makinde, P.Y. Mhone, "Heat Transfer to MHD Oscillatory Flow in a Channel filled with porous medium", Rom. Journ. Phys. Vol. 50, 2005, pp. 931-938.

[7] A. Ogulu, A. R. Bestman, "Deep Heat Muscle Treatment - A Mathematical Model", pp. 3-16.